

Q1 ANALYSIS OF SEDIMENTATION PATTERNS USING LEAD-210 (10 pts)

Part 1. Calculation of Tracer Activity and Relative Efficiency (2.1 pts)

1.1 Decay constant, λ :

EXCEL EQUATION =LN(2)/B#

 $\lambda = \ln(2) / t_{1/2}$

1.1	lsotope	λ		
	Pb-210	0.0311	У ⁻¹	0.1 pt
	Po-210	0.0050	d ⁻¹	0.1 pt
	Po-208	0.2366	У ⁻¹	0.1 pt

1.2 Tracer concentration after plating (in Bq/ml):

EXCEL EQUATION =1.33*EXP(-((B#-\$G\$3)/365.25)*\$I\$10)

G3 = tracer date, 1 = λ_{Po208}

$$t = \frac{(plating \ date - tracer \ date) \ days *}{365.25 \ days/y}$$

 $A_0 = 1.33 \ Bq/ml$ [given]

 $A_t = A_0 \exp(-\lambda_{Po208} t)$

* if dates are subtracted directly, in excel the answer is expressed as days, so it must first be converted to y (since λ is in y⁻¹)

Tracer concentration after plating (in dpm/ml):

EXCEL EQUATION =C#*60

$$1Bq = 1\frac{disintegration}{s} \times \frac{60 \, s}{min}$$

Average: EXCEL EQUATION =AVERAGE(C3:C20) or =AVERAGE(D3:D20)

Standard Deviation: EXCEL EQUATION = STDEV.P(C3:C20) or =STDEV.P(D3:D20)

1.2	Average: 0.5679 Bq/ml or 34.0718 dpm/ml	0.3 pt
	Standard Deviation: 0.0028 Bq/ml or 0.1664 dpm/ml	0.2 pt





1.3a. Activity concentration: **EXCEL EQUATION =G#/(E#/60)-F#**

 $A_{experimental} = \frac{\text{counts}}{\text{count time / 60}} - A_{bkg}$

 $1 \ count = 1 \ disintegration$

1.3a	Depth range, cm	Po-208 Counts	Experimental Tracer Activity, dpm	0.2 pt
	0 - 1	1251	0.9068	each
	29-30	1079	0.7473	cucii

1.3b Calculated tracer activity: Since time elapsed between plating and counting date, recalculate activity of Po-208 tracer to match when using the experimental data

EXCEL EQUATION =B#*EXP(-(\$O\$5)*(C3-D3)/365.25)*0.2

\$O\$5 = λ_{Po208}

 $A_t = A_0 \exp(-\lambda_{Po208} t)$

t = *counting date* – *plating date* [must be in y]

Recalculated tracer activity (in dpm) = $A_t \times 0.2 \text{ ml}$

Relative efficiency: EXCEL EQUATION =I#/H#

Relative Efficiency = $\frac{A_{experimental}}{A_{calculated}}$

Average relative efficiency: EXCEL EQUATION =AVERAGE(J3:J20)

Average Relative Efficiency, $RE_{ave} = \frac{\left(\sum_{i=1}^{18} Relative Efficiency_i\right)}{18}$

Standard deviation: EXCEL EQUATION = STDEV.P(J3:J20)

1.3b	Average relative efficiency: 0.1262	0.3 pt
	Standard Deviation: 0.0323	0.2 pt

Part 2. Lead-210 Determination (2.3 pts)

2.1a. Po-210 Activity: EXCEL EQUATION =(H#/(F#/60)-G#)/C#*EXP(\$S\$4*((D#-E#))) Or =(H#/(F#/60)-G#)/C#/EXP(-\$S\$4*((D#-E#)))

 $S^4 = \lambda_{Po210} = 0.005$





$$A_0 = (A_t - A_{bkg}) \times RE \times \exp(\lambda_{Po210}t)$$

$$A_0 = \frac{(A_t - A_{bkg}) \times \text{RE}}{\exp(-\lambda_{Po210}t)}$$

t = *counting date* – *plating date* [must be in d]

Total Pb-210: EXCEL EQUATION =(I#*EXP((\$S\$3)*((D#-\$Q\$7)/365.25))) $S^3 = \lambda_{Pb210}$, $Q^7 =$ sampling date

Since [Po-210] = [Pb-210],

 $A_0 = (A_t) \exp(\lambda_{Pb210} t)$

t = *plating date* – *sampling date* [must be in y]

Relative error:

EXCEL EQUATION =2*SQRT((1/H#)+(1/'(1.3) Relative Efficiency'!G#))

Relative Error =
$$2 \operatorname{sqrt}(\frac{1}{C_x} + \frac{1}{C_t})$$

in ± Bq/kg, Relative Error × Total Pb210 $\left(in \frac{Bq}{kg}\right)$

2.1 Depth range, cm	Total Po-210 Activity, Bq/kg	Total Pb-210 Activity, Bq/kg	Error, ± Bq/kg
0 – 1	122.6525	124.4292	9.6450
29 - 30	86.4196	87.6715	7.9870

2.2









59.5	0.008 closest to 0	
69.5	0.081	
79.5	0.113	
89.5	0.122	
99.5	0.086	
109.5	0.124	
119.5	-0.029	
129.5	0.106	
139.5	0.281	
147.5	0.713	
Supported Pb – 2	$10 = Average \ of \ conc. from \ 59.5 \ to \ 149.5$	=AVERAGE(B10:B20)
Standard Deviati	on	=STDEV.P(B10:B20)
2.2 Suppor	rted Pb-210: 13.6352 Bq/kg	0.8 pt
Standa	rd Deviation: 4.4691 Bq/kg	0.3 pt

Part 3. Lead-210 Geochronology (4.6 pts)

3.1a EXCEL EQUATION ='(2.2) Pb-210 Plot'!B3-'(2.2) Pb-210 Plot'!\$M\$4



'(2.2) Pb-210 Plot'!\$M\$4 = supported Pb-210

The slope, m of the linear fit is calculated as follows:

$$m = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - (\sum x)^2} = \frac{4(63.337) - (31.0)(8.245)}{4(473) - (31.0)^2} = -0.0024$$

While the y-intercept, *b* is obtained from:





$$b = \frac{\sum y - m\sum x}{n} = \frac{8.245 - (-0.0024)(31.0)}{4} = 2.0798$$

3.1b	Depth (x)	LogPbEx-210 (y)	ху	x ²	
	0.5	2.0445	1.0223	0.2500	
	1.5	2.0957	3.1436	2.2500	
	9.5	2.0866	19.8229	90.2500	
	19.5	2.0179	39.3482	380.2500	
	Σx = 31.0	Σy = 8.245	Σxy = 63.337	$\Sigma x^2 = 473.0000$	0.1 pt/oach
					U. I pt/each
	m = -0.0024				10tal = 0.7
	b = 2.0798				pts
	Linear equation	n: y = -0.0024x + 2.	0798		

3.1	X	у	ху	x ²	
d	19.5	2.0179	39.3482	380.2500	
	29.5	1.8694	55.1486	870.2500	
	39.5	1.7065	67.4068	1560.2500	
	49.5	1.4557	72.0579	2450.2500	
	59.5	0.9812	58.3809	3540.2500	0.1 nt/eac
	Σx = 197.5	Σy = 8.0307	Σxy = 292.3424	$\Sigma x^2 = 8801.2500$	h
	m = -0.0249 b = 2 5885				Total = 0.7
	b = 2.5885 Linear equation: y = -0.0249x + 2.5885				

3.1e EXCEL EQUATION =(R3)/(-2.303*E1)

$$S = -\lambda \left(\frac{x_2 - x_1}{\ln\left(\frac{A_1}{A_2}\right)} \right)$$
$$slope = \frac{rise}{run} = \frac{\log\left(\frac{A_2}{A_1}\right)}{x_2 - x_1} = \left(\frac{1}{2.303}\right) \left(\frac{\ln\left(\frac{A_2}{A_1}\right)}{x_b - x_a}\right)$$





$$2.303 \times slope = \left(\frac{\ln\left(\frac{A_2}{A_1}\right)}{x_b - x_a}\right)$$

$$S = -\lambda_{Pb210} \left(\frac{1}{2.303 \times slope} \right) = 0.5427 \ cm/y$$

3.1e	Sedimentation Rate: 0.5427 cm/y	0.8 p [.]	t
	Acceptable answers: $0.4706 \text{ cm/y} < S < 0.6409 \text{ cm/y}$		

EXCEL EQUATION =DATE(2009,4,29)-LN(B6/B8)/(R3)*365.25 3.2

$$\ln\left(\frac{A_{1}}{A_{2}}\right) = \lambda_{Pb210}(t_{2} - t_{1})$$

$$age = t_{2} - t_{1} = -\ln\left(\frac{A_{19.5cm}}{A_{39.5cm}}\right) / (\lambda_{Pb210})$$

3.2 23 years 0.8 pt

Part 4. Cesium-137 Validation (1.0 pt)

4.1a



Sedimentation rate
$$=$$
 $\frac{(49.5 - 39.5) cm}{(1986 - 1963)y} = 0.435 cm/y$

4.1 0.4348 cm/y





Sample	Pb-210	Cs-137
Core-1	0.5427	0.4348
Core-2	0.6213	0.5205
Core-3	0.5206	0.4309
Core-4	0.4911	0.4715
Core-5	0.5706	0.5304
Core-6	0.5516	0.5087
х	0.5497	0.4828
S	0.0406	0.0398

$$t_{calc} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$t_{calc} = \frac{0.5497 - 0.4828}{\sqrt{\frac{0.0406^2}{6} + \frac{0.0398^2}{6}}} = 2.8812$$
$$t_{tab,95\%} = 2.5706$$

$$t_{calc} > t_{tab}$$
, significant difference!

4.2 (a) 2.8812 (b) No

0.3 pt 0.2 pt 1st INSO International Nuclear Science Olympiad



Q2 SHIELDING ACTIVATED MATERIAL (10 pts)

Part 1. Data Collection (1.5 pts)

1.1 From JANIS, we can retrieve the values of σ for different energies, for the case of the problem, we have $E_{th} = 0.025$ eV. For the case of this energy, JANIS does not show the cross-section for the specific energy. For Cr-50 we have,

$$\sigma(E_{th} = 0.023975) = 15.824$$

 $\sigma(E_{th} = 0.0253) = 15.405.$

We can use linear interpolation to get the σ for the specific energy.

$$\sigma = \sigma_1 + (E - E_1) \frac{\sigma_2 - \sigma_1}{E_2 - E_1}$$

 $\sigma(0.025 \ eV) = 15.824 + (0.025 - 0.023975) \frac{15.405 - 15.824}{0.0253 - 0.023975}$

 $\sigma(0.025 eV) = 15.500$

	r			
1.1	lsotope	Cross-Section (barns)	Natural Abundance, (%)	of
	Cr-50	15.500	4.345	1.0 pts
	Mn-55	13.361	100	0.125
	Fe-58	1.157	0.282	/item
	Co-59	37.413	100	
				-

1.2 From JANIS, you will arrive to

1.2	lsotope	Half-life (sec)	of
	Cr-51	2.393×10^{6}	0.5 pts
	Mn-56	9.288×10^{3}	0.125
	Fe-59	3.844×10^{6}	/item
	Co-60	1.663 × 10 ⁸	

Part 2. Neutron Activation (2.25 pts)

2.1 In order to get the prompt activity, we need to calculate many values. First, we need to calculate for the respective masses of the elemental composition in the sample. With a mass of 350 g and the weight fraction of each element given in the table, we can get,





Element	Weight Percent (%)	Mass (g)
Cr	8.0	(350)(8%) = 28
Mn	14.0	(350)(14%) = 49
Fe	68.85	(350)(68.85%) = 240.975
Со	1.0	(350)(1.0%) = 3.5

Next is to convert the units of the cross-section from barns to $cm^2.$ We use 1 barn = $1\times 10^{24}\,cm^2$.

lsotope	Cross-section (barns)	Cross-section (cm ²)
Cr-50	15.500	1.550×10^{-23}
Mn-55	13.361	1.336×10^{-23}
Fe-58	1.157	1.157×10^{-24}
Co-59	37.413	3.741×10^{-23}

We also need the total number of target atoms of the target nuclide that can be calculated using

$$N = \frac{mf_i N_A}{M}$$

For Mn-55:

$$N_{Mn-55} = \frac{(49)(100\%)(6.02214 \times 10^{23})}{54.938} = 5.371 \times 10^{23}$$

Using the values of the mass m, abundance f_i , Avogadro's Number and the molar mass M, we get

lsotope	N (total number of target atoms of the target nuclide)	
Mn-55	5.371×10^{23}	
Fe-58	7.064×10^{21}	
Co-59	3.577×10^{22}	

Now to calculate for the prompt activity, we need to use the total number of target atoms of the target nuclide, thermal cross section in cm², and thermal neutron flux. Since the flux is fully thermalized, the equation needed is

$$A_{\infty} = N\phi_{th} \,\sigma_{th}$$





The value of the thermal flux is $\phi_{th} = 2.460 \times 10^{14} \text{ cm}^{-2} \text{s}^{-1}$. For Mn-55, we get the prompt activity in Bq as

 $A_{\infty(Mn-55)} = (5.371 \times 10^{23})(1.336 \times 10^{-23})(2.460 \times 10^{14}) = 1.765 \times 10^{15}$

Target nuclei	N (total number of target atoms of the target nuclide)	Thermal Neutron Cross Section (cm²)	Saturation Activity (Bq)
Mn-55	5.371×10^{23}	1.336×10^{-23}	1.765×10^{15}
Fe-58	7.064×10^{21}	1.157×10^{-24}	2.011×10^{12}
Co-59	3.577×10^{22}	3.741×10^{-23}	3.292×10^{14}

For the next part, we can calculate for the decay constant of the activated product using the equation $\lambda = \frac{ln(2)}{t_{1/2}}$.

For Mn-56,

 $\lambda = \frac{ln(2)}{9.288 \times 10^3} = 7.463 \times 10^{-5}$

Activated Product	Half-life (sec)	Decay Constant
Mn-56	9.288×10^{3}	7.463×10^{-5}
Fe-59	3.844×10^{6}	1.803×10^{-7}
Co-60	1.663×10^{8}	4.167×10^{-9}

Lastly, we need to calculate for the activity after the 5-hour (18000 s) irradiation using the equation

$$A(t) = A_{\infty} \left(1 - e^{-\lambda t} \right)$$

So, for the example of the Mn-56 nuclei, we have

$$A(18000) = 1.765 \times 10^{15} (1 - e^{(-7.463 \times 10^{-5})(18000)}) = 1.304 \times 10^{15} \text{ Bq}$$

The following table shows the result for all the other nuclei,

Activated Product	Prompt Activity after 5 hours (Bq)	
Mn-56	$1.304 imes10^{15}$	
Fe-59	6.516×10^{9}	
Co-60	2.469×10^{10}	





Ζ.Ι	Activated Product	Prompt Activity after 5 hours (Bq)	1.5 pts
	Mn-56	$(1.304 \pm 0.001) \times 10^{15}$	0.5
Í	Fe-59	$(6.516 \pm 0.002) \times 10^9$	/item
	Co-60	$(2.469 \pm 0.001) \times 10^{10}$	

Using the given formula in the problem,

$$\varphi = \frac{pA}{4\pi r^2}$$

we can get the flux for each gamma energy required for the problem, namely, 0.847 keV, 1.099 MeV, and 1.252 MeV. We are also given the distance to a detection point of 0.5 meters away and the assumption that no gamma radiation attenuation and scattering is present. The following table presented shows the corresponding values of p for each energy of interest.

Main Activation Product	Gamma Radiation Emitted	Emission Probability, p	
	(MeV)	(%)	
Mn-56	0.847	98.85	
	1.810	26.9	
Fe-59	1.099	56.5	
	1.292	43.2	
Co-60	1.252	199.83	

For gamma energy of 0.847 MeV:

$$\rho = \frac{(0.9885)(1.304 \times 10^{15})}{(4)(\pi)(500)^2} = 4.105 \times 10^8$$

For gamma energy of 1.099 MeV:

$$\varphi = \frac{(0.565)(6.516 \times 10^9)}{(4)(\pi)(500)^2} = 1.171 \times 10^3$$

For gamma energy of 1.252 MeV:

 $\phi = \frac{(1.9983)(2.469 \times 10^{10})}{(4)(\pi)(500)^2} = 1.570 \times 10^4$

Gamma Energy	Flux (#/cm ² -sec)
0.847 MeV	$4.103 imes 10^{8}$
1.099 MeV	1.172×10^{3}





 $1.570 imes 10^4$

2.2	Gamma Energy	Flux (#/cm ² -sec)	0.75 pts
	0.847 MeV	$(4.103 \pm 0.002) \times 10^8$	0.25
	1.099 MeV	$(1.172 \pm 0.001) \times 10^3$	/item
	1.252 MeV	$(1.570 \pm 0.001) \times 10^4$	

Part 3. Radiation Shielding (3.0 pts)

The mass attenuation coefficients (μ/ρ) , linear attenuation coefficients (μ) , and the mean free paths (mfp) of the three containers at three gamma energies obtained from the EpiXS software.

	Alloy 1			Alloy 2			Alloy 3		
Gamma Energy	μ/ρ (cm^2 g^(-1))	μ(cm^(-1))	MFP (cm)	μ/ρ (cm^2 g^(-1))	μ(cm^(-1))	MFP (cm)	μ/ρ (cm^2 g^(-1))	μ(cm^(-1))	MFP (cm)
0.847 MeV	0.06509	0.51094	1.95718	0.06548	0.52839	1.89256	0.06522	0.51588	1.93845
1.099 MeV	0.05710	0.44824	2.23094	0.05743	0.46343	2.15781	0.05721	0.45255	2.20971
1.252 MeV	0.05345	0.41948	2.38390	0.05374	0.43367	2.30588	0.05354	0.42351	2.36125

3.1 Alloy 2 is the most effective container for shielding gamma energy of 1.099 MeV - highest μ/ρ and μ , & smallest mfp (from Table above).

3.1	(a) $0.05743 \text{ cm}^2 \text{g}^{-1}$	0.3 pt
	(b) 0.46343 cm^{-1}	0.3 pt
	(c) 2.15781 cm	0.3 pt

3.2 For 1.252 MeV:

Alloy 1

$$I = I_0 e^{-\mu x} = (1.570 \times 10^4) (e^{(-0.41984)(2.1261)}) = 6435.337 \text{ photons/cm}^2 \cdot \text{s}$$

Alloy 2

$$I = I_0 e^{-\mu x} = (1.570 \times 10^4) (e^{(-0.43367)(2.0562)}) = 6436.226 \text{ photons/cm}^2 \cdot \text{s}^2$$





Alloy 3

$$I = I_0 e^{-\mu x} = (1.570 \times 10^4) (e^{(-0.42351)(2.1058)}) = 6435.592 \text{ photons/cm}^2 \cdot \text{s}$$

The Alloy 1 container offers the best protection by providing the lowest transmitted gamma radiation flux. When the activated sample is stored inside the Alloy 1 container, the transmitted gamma radiation flux is 6436.698 $\frac{\text{photons}}{\text{cm}^2 \cdot \text{s}}$.

3.2
$$I = 6435.337 \text{ photons}/_{\text{cm}^2 \cdot \text{s}}$$
 1.0 pt

3.3 To calculate the ratio of the final/transmitted flux from the initial flux for Alloy 3 container and for all gamma energies, we first combine the total flux of all three energies.

$$I_{initial} = 4.103 \times 10^8 + 1.172 \times 10^3 + 1.570 \times 10^4 = 4.103 \times 10^8$$

We now calculate for each final intensity for all gamma radiation energies as it passes through alloy 3.

$$\begin{split} I_{final_{E1}} &= 4.103 \times 10^8 e^{(-0.51588)(2.1058)} = 1.385 \times 10^8 \\ I_{final_{E2}} &= 1.172 \times 10^3 e^{(-0.45255)(2.1058)} = 4.519 \times 10^2 \\ I_{final_{E1}} &= 1.570 \times 10^4 e^{(-0.42351)(2.1058)} = 6.436 \times 10^3 \end{split}$$

And the total final flux is

$$I_{final} = 1.385 \times 10^8 + 4.517 \times 10^3 + 6.437 \times 10^3 = 1.385 \times 10^8$$

Calculating the ration, we get

$$\frac{I_{final}}{I_{intial}} = \frac{1.385 \times 10^8}{4.103 \times 10^8} = 0.337$$

3.3 0.337

1.1 pt

Part 4. Shielding Optimization (3.25 pts)

4.1

With have assumption that a radiation worker is expected to handle activated samples for a total of 800 hrs a year and also the five-year averaged dose limit of 20 mSv/year, we can convert the dose limit in terms of $\mu Sv/hr$ as follows,





$$20\frac{\mathrm{m}Sv}{\mathrm{year}} \cdot \frac{1}{800}\frac{\mathrm{year}}{\mathrm{hours}} \cdot \frac{1000\,\mathrm{\mu}Sv}{1\,\mathrm{m}Sv} = 25\frac{\mathrm{\mu}Sv}{\mathrm{hr}}$$

4.1	$25.00 \frac{\mu Sv}{m}$	0.25 pt	t
	20.00 hr		

a) We start with the combination of equations to arrive at the desired gamma dose. We have

$$I = I_{\nu}Be^{-\mu x}$$

and

$$B = Ae^{-\alpha_1\mu x} + (1 - A)e^{-\alpha_2\mu x}$$

Combining both equations, we get

$$I = I_{\gamma} (A e^{-\alpha_1 \mu x} + (1 - A) e^{-\alpha_2 \mu x}) e^{-\mu x}.$$

From Part 2.2, the highest contributor is the gamma source with an energy of 0.847 MeV with a gamma intensity of $I_{\gamma} = 4.103 \times 10^8$. We first calculate of the values of A, α_1 , and α_2 for the gamma energy of 0.847 MeV.

For A:

$$A = 1.677 + (0.847 - 0.5)\frac{(2.840 - 1.677)}{(1.0 - 0.5)} = 2.484$$

For α_1 :

$$\alpha_1 = -0.03084 + (0.847 - 0.5) \frac{(-0.03503 - (-0.03084))}{(1.0 - 0.5)} = -0.0337$$

For α_2 :

$$\alpha_2 = 0.30941 + (0.847 - 0.5) \frac{(0.13486 - 0.30941)}{(1.0 - 0.5)} = 0.188$$

In order to get to a dose value, we need to multiply the calculated gamma flux by the flux-to-dose conversion for the energy of interest.

Gamma Energy (MeV)	Conversion Coeff. (µSv/hr)/(n/cm²-s)
0.662	1.085
0.800	1.310
1.000	1.625
1.117	1.800





2.100

For gamma energy of 0.847 MeV:

$$1.310 + (0.847 - 0.8)\frac{(1.625 - 1.310)}{(1.0 - 0.8)} = 1.384$$

The final calculated dose can now be expressed as:

Calculated Dose = $(1.384)(4.103 \times 10^8)(2.484e^{0.0337\mu x} + (1 - 2.484)e^{-0.188\mu x})e^{-\mu x}$.

We are given the range of values from $\mu x = 17$ to 19. By plotting both the desired gamma dose from Part 4.1 and the calculated gamma dose from the previous equation at the said range, we will get,



b) From the plot, we can see that both lines intersect at around $\mu x = 18.5$. We may test the gamma dose for $\mu x = 18.47$ and we get.

 $\begin{aligned} \textit{Calc.Dose} &= (1.384)(4.103 \times 10^8) \big(2.484 e^{(0.0337)(18.47)} + (1 - 2.484) e^{(-0.188)(18.47)} \big) e^{-18.47} \\ &= 24.7958 \end{aligned}$

We can then calculate for the thickness of the Pb block with the use of this value of μx and the attenuation coefficient of Pb, which can be retrieved from EPIXS for gamma energy of 0.847 MeV as $\mu = 0.944$





Thickness = $\frac{\mu x}{\mu} = \frac{18.47}{0.944} = 19.57 \text{ cm}$

